

Black-box passive macromodeling in electronics: trends and open problems

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Editor: E. Jan W. ter Maten, Wuppertal, Germany  
Assistant: Piotr Ptak, Lappeenranta, Finland

**Editor's address:**

Bergische Universität Wuppertal  
FB-C, Applied Mathematics and Numerical Analysis  
Gaußstrasse 20  
D-42119 Wuppertal  
Germany  
email: *Jan.ter.Maten@math.uni-wuppertal.de*

**Distribution:**

ECMI Secretariat

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Department of Mathematics & Computer Science  
CASA TU Eindhoven  
PostBox 513, Den Dolech 2, MF 7.140 NL-5600 MB Eindhoven, The Netherlands  
Phone +31 40 24 78389/72753; Fax: 42489  
email: *e.j.w.ter.maten@TUE.nl*

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Joost Rommes

Mentor Graphics

110 Rue Blaise Pascal, Viséo B, Inovallée Montbonnot,

38330 Montbonnot

France

<http://sites.google.com/site/rommes>

[joost\\_rommes@mentor.com](mailto:joost_rommes@mentor.com)

## Black-box passive macromodeling in electronics: trends and open problems

### Abstract

Design and verification flows in the electronics industry are relying more and more on behavioral models of components, electrical interconnects, and subsystems. Such models are often derived from tabulated frequency responses obtained via direct measurements or through electromagnetic field solvers. Model extraction from this data involves a mix of system identification and approximation in the complex frequency domain. This problem becomes difficult or badly scalable due to the presence of passivity constraints, which must be enforced during model extraction. We review recent trends to deal with this complexity, and related open issues.

### Electrical interconnects

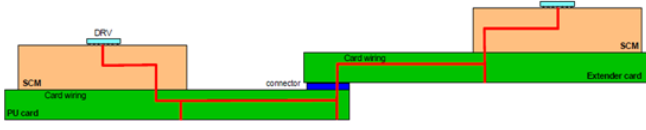
Let us consider a modern electronic system, such as a server for high-performance computing (Fig. 2) or a smartphone. These objects include a complicated network of wires routed through chips, packages and boards, that provide electrical connectivity between all system parts (Figs. 3 and 4). Some of these wires are responsible for delivering power in form of a constant supply voltage, some others are responsible for delivering high-speed digital data signals to possibly billions of interconnected components.

The fact that so many electrical interconnects must coexist in close proximity is one of the major challenges in electronic design. This is due to unwanted electromagnetic interactions that inevitably take place within the system, leading to parasitic couplings between conductors that by design are supposed to be electrically separated. Such couplings are observed in form of noise spreading in the system. If proper countermeasures are not taken, this noise may be so large to disrupt system behavior, leading to malfunctioning.

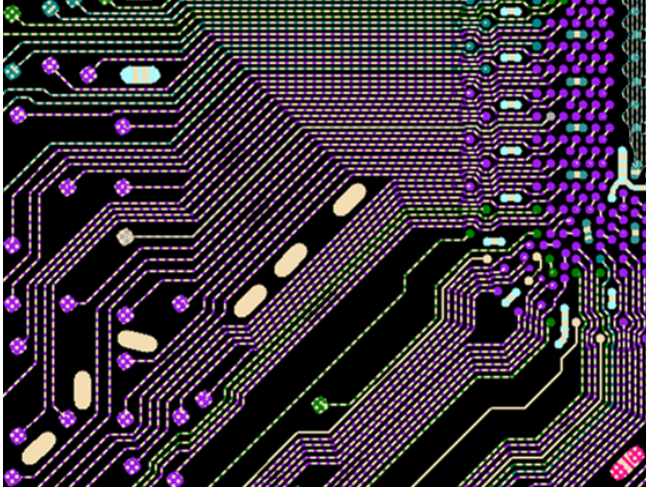
### Modeling approaches

The first step for understanding noise generation and propagation through the system is the solution of an electromagnetic field problem. Several approaches exist, based on time-domain or frequency-domain differential or integral forms of Maxwell's equations. The underlying electromagnetic system is linear, and the main challenge is in the extreme complexity of geometry, with fine details over a wide range of scales, and especially non-ideal material properties (e.g., skin and proximity effects that cause a nonuniform current density flow within a single conductor cross-section). Although research is ongoing to improve the capacity of field solver engines, design flows in industry rely on commercial solvers, which invariably provide their results in forms of tabulated frequency responses of the system over a finite bandwidth, and at a finite number of interface ports where input and output signals are defined. The number  $K$  of available frequency values often exceeds several thousands or tens of thousands, whereas the number  $P$  of input/output ports can reach several hundreds or more. Most often the frequency responses are defined and computed in the Scattering representation, with inputs being

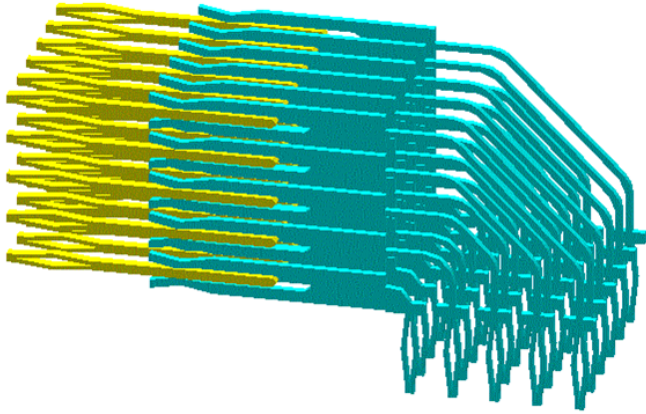
incident power waves into the structure, and outputs being the corresponding reflected power waves. This justifies the common denomination “S-parameter block”.



**Figure 2:** Schematic illustration of a server, highlighting a chip-to-chip interconnect link routed through packages, boards, and connectors.



**Figure 3:** One layer (portion) of an electronic package.



**Figure 4:** A CAD model of a high-speed connector.

Signal and power integrity verification requires repeated cycles of time-domain simulations of several interconnected S-parameter blocks, terminated at their interfaces by nonlinear and dynamic device models representing transistors or groups of transistors. Such system-level simulations are exceedingly complex and require major computing resources, forming a major bottleneck in product design flows.

The so-called *passive macromodeling* strategy provides a convenient approach to reduce this complexity. We process each individual S-parameter block, and we extract a corresponding reduced-order state-space model

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}\quad (7)$$

by enforcing the fitting condition  $\mathbf{H}(j\omega_k) \approx \hat{\mathbf{H}}_k$  over the available frequency samples  $\omega_k$ ,  $k = 1, \dots, K$ , where

$$\mathbf{H}(s) = \mathbf{C}(s\mathbb{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (8)$$

is the transfer function of (7), and where  $\hat{\mathbf{H}}_k \in \mathbb{C}^{P \times P}$  represents the frequency response obtained from the electromagnetic solver at frequency  $\omega_k$ . The computation of state-space matrices  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$  is often performed by rational curve fitting (e.g., using the so-called “Vector Fitting” algorithm [9],[5] followed by a state-space realization, but there exist approaches (e.g., Löwner matrix interpolation [11]) that generate directly the state matrices. Both these approaches are efficient and can handle large input datasets. Once a model in form (7) is available, its inclusion as a component in standard circuit simulation environment such as SPICE is straightforward.

### Passivity conditions

The above-described model construction is not complete and likely to fail in production environments. In fact, the model must fulfill some fundamental physical consistency properties, such as (asymptotic) stability and more generally passivity. Two are the main reasons: first, electrical interconnects are passive structures (they are unable to generate energy), so any model that intends to represent them must be passive to be realistic; second, if a model is not passive, when interconnected with other (even passive) models, it may give rise to instabilities, leading to transient simulations that blow up for late time [7]. All modeling efforts would then be wasted.

For scattering representations, passivity requires that the model transfer function  $\mathbf{H}(s)$  is *bounded real* [1]. When the state-space matrices are real-valued and all eigenvalues of  $\mathbf{A}$  have a strictly negative real part (these two conditions are easily enforced in the fitting phase), bounded realness holds if  $\mathbf{H}(s)$  is an element of the Hardy space  $\mathcal{H}_\infty$ , with

$$\|\mathbf{H}\|_{\mathcal{H}_\infty} \leq 1, \quad (9)$$

or equivalently when

$$\sigma_1(j\omega) = \max \sigma(\mathbf{H}(j\omega)) = \|\mathbf{H}(j\omega)\| \leq 1 \quad \forall \omega \in \mathbb{R}, \quad (10)$$

where  $\sigma(\cdot)$  denotes the set of singular values of its matrix argument. An alternative passivity condition is provided by the *Bounded Real Lemma (BRL)*, which states that system (7) is passive if and only if

$$\begin{pmatrix} \mathbf{A}^\top \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{C}^\top \mathbf{C} & \mathbf{P} \mathbf{B} + \mathbf{C}^\top \mathbf{D} \\ \mathbf{B}^\top \mathbf{P} + \mathbf{D}^\top \mathbf{C} & -(\mathbb{I} - \mathbf{D}^\top \mathbf{D}) \end{pmatrix} \leq 0 \quad (11)$$

for some matrix  $\mathbf{P} = \mathbf{P}^\top > 0$ . Finally, provided that  $\|\mathbf{D}\| < 1$ , passivity holds if the Hamiltonian matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{D}^\top \mathbf{C} & \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^\top \\ -\mathbf{C}^\top \mathbf{S}^{-1} \mathbf{C} & -\mathbf{A}^\top - \mathbf{C}^\top \mathbf{D}\mathbf{R}^{-1}\mathbf{B}^\top \end{pmatrix} \quad (12)$$

where  $\mathbf{R} = \mathbb{I} - \mathbf{D}^\top \mathbf{D}$  and  $\mathbf{S} = \mathbb{I} - \mathbf{D}\mathbf{D}^\top$ , does not have purely imaginary eigenvalues.

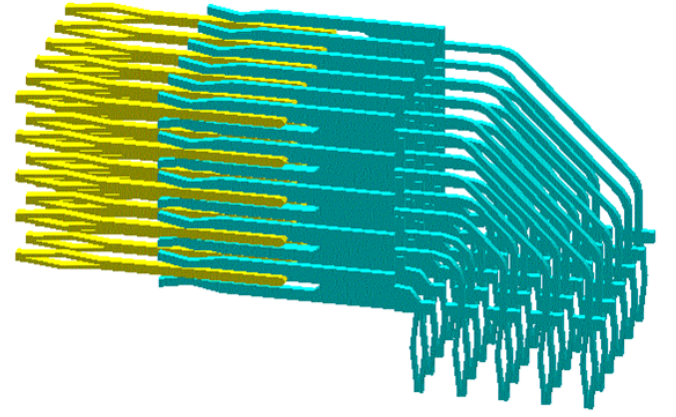
## Passivity enforcement

Direct enforcement of any of the above passivity constraints in the model identification stage is not practical for the large-scale models that are found in electronic applications. Therefore, common modeling flows involve a first model extraction without passivity constraints, followed by a second perturbation stage where passivity is enforced. The state-space matrices of the initial model are perturbed, and the nearest passive model is sought for by minimizing a suitable cost function (e.g., the minimum perturbation of the impulse response in  $\mathcal{L}_2$  sense). Various algorithms have been proposed [2],[4],[12],[10],[6], each based on a particular passivity constraint in form (9)–(12). We believe that there is still significant margin for improvement.

- If the  $\mathcal{H}_\infty$  norm constraint (9) is used, the perturbation-based passivity enforcement problem is convex, and the optimal solution will be found. However, since the  $\mathcal{H}_\infty$  norm is a nonsmooth function of the decision variables, one cannot use gradient-based descent schemes and has to resort to subgradient or localization methods [2]. These are well-known to require many iterations with a very slow convergence rate. So, even if optimal, such schemes are impractical.
- Similar difficulties arise when using the BRL constraint (11). Also in this case passivity enforcement can be cast as a convex optimization, but the constraint is here represented as a Linear Matrix Inequality where also the auxiliary matrix  $\mathbf{P}$  is an unknown [4]. The main difficulty in applying off-the-shelf semidefinite programming algorithms is the excessive memory requirement. So, also this approach is only viable for small-scale academic examples that are far from real-world applications.
- Suboptimal schemes based on local passivity constraints are widely used [12],[10]. For instance, one may enforce (10) only at a finite number of frequencies  $\omega_n$ ,  $n = 1, \dots, N$ , only for those singular values such that  $\sigma_i(j\omega_n) > 1$ . This perturbation requires iterations and is not globally convex, so that it may happen that the scheme does not converge to a solution. Even if found, this solution may not be optimal.
- Another suboptimal scheme that is widely used is based on perturbation of the imaginary eigenvalues of the Hamiltonian matrix  $\mathbf{M}$  in (12). Using a first-order Hamiltonian matrix perturbation, one constrains these eigenvalues to move off the imaginary axis, thus achieving model passivity [6]. Also this approach requires iterations and is not globally convex.

In summary: optimal schemes are too heavy, and suboptimal schemes may not converge or may not lead to sufficiently accurate models [8]. Therefore, fundamental research efforts both in formulation and in algorithm development are still needed to keep the pace of technology and manufacturing advancements. It is remarkable that it is easier to build highly complex electronic systems than producing accurate and scalable models for their parts.

## Modeling distributed systems



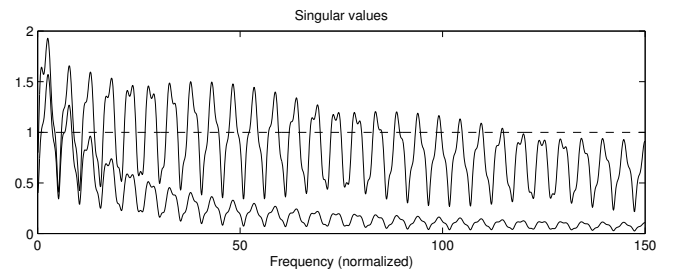
**Figure 5:** Frequency response of an electrically long interconnect.

High-speed interconnects carry signals characterized by a spectrum that extends to very high frequencies. The larger the frequency, the smaller is the characteristic wavelength of the electromagnetic field associated with the signals. When this wavelength is smaller than the physical size of the system, the effects due to the finite propagation speed of the electromagnetic field become visible in the system responses in terms of delays, and the macromodeling process is more challenging.

An accurate representation of propagation effects requires a suitable inclusion of delay terms in the model. For instance, a possible model structure that achieves this goal is

$$\mathbf{H}(s) = \left( \sum_{m=1}^M \mathbf{C}_m e^{-s\tau_m} \right) (s\mathbb{I} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}, \quad (13)$$

where  $\tau_m > 0$  are the delays. As an example, Fig. 5 compares a frequency response of a passive model with structure (13) to the corresponding “true” system response for a simple electrically long interconnect. One can note that the phase is characterized by fast oscillations, induced by the presence of the delay terms  $e^{-s\tau_m}$ . If a standard lumped model (7) were used, without an explicit extraction and inclusion of these terms in the model structure, the number of poles that would be required for an accurate representation of the frequency response would be exceedingly large and impractical.



**Figure 6:** Frequency-dependent singular values of a (non-passive) delay-rational model for a simple two-port interconnect.

For these delay-rational macromodels, another layer of complexity adds to the already challenging passivity enforcement

problem. For instance, the Hamiltonian matrix (12) becomes frequency-dependent, and the associated eigenproblem reads

$$\mathbf{M}(s)\mathbf{w} = s\mathbf{w}, \quad (14)$$

where  $\mathbf{M}(s)$  is a linear combination of (incommensurate) delay terms [3]. The purely imaginary solutions  $s_k = j\omega_k$  of (14) correspond to the frequencies  $\omega_k$  at which one of the singular values of the model transfer function cross or touch the passivity threshold  $\sigma = 1$ . The number of these eigenvalues is not upper bounded, as in the delayless case, by the size of  $\mathbf{M}(s)$ : the quasi-periodicity of  $\mathbf{M}(s)$  is in fact responsible for a possibly very large number of such eigenvalues, as depicted in Fig. 6 for a simple test case.

Very little results are available to reliably check and enforce model passivity in this delayed case. A complete search of all imaginary eigenvalues, e.g., via rational Krylov solvers, is in principle feasible but likely to be very time consuming. In addition, deployment of a perturbation scheme for all such imaginary eigenvalues might even be unnecessary, given their strong correlation induced by quasi-periodicity. For these reasons, passive macromodeling of distributed interconnect macromodels including delay terms is considered to be an interesting opportunity for future research.

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**Stefano Grivet-Talocia**

Department of Electronics and Telecommunications  
Politecnico di Torino  
Corso Duca degli Abruzzi 24, 10129 Torino  
Italy  
[Stefano.Grivet@polito.it](mailto:Stefano.Grivet@polito.it)